



*This brainteaser was written by Derrick Niederman.*

“Mom, look at that license plate,” Will said.

“What about it?” his mother asked. It didn’t seem unusual to her. The plate consisted of two sets of three digits, with the state logo between the sets.



Will said, “All six digits are different. And when you multiply the first three digits, you get the same product as when you multiply the last three digits.”

“So you do,” his mother said. “How many plates like that do you suppose there are?”

“Well, that’s the cool part,” Will replied. “The number of plates like that is equal to the product of the first three digits.”

What license plate might Will have seen?



**Solution: One possibility is 819-634.**

The product of the first three digits equals 72, the same as the product of the last three digits. Note that any rearrangement of these two groups of numbers (such as 189-463) will also work. There are six rearrangements of the first three digits, another six rearrangements of the last three digits, and the two sets of digits can be interchanged, for a total of  $6 \times 6 \times 2 = 72$  possible solutions.

Why these particular six digits? Well, it's easy to see that 0 cannot occur in the license plate, since its appearance would force one of the products to be 0, while the other product would be non-zero. And 5 and 7 cannot appear, because only one of the two products would have a prime factor of 5 or 7. Specifically, 2 and 3 are the only primes that can appear, because each one has multiples of itself among the remaining digits.

Note that the license plate 236-419 and all its rearrangements also satisfy the condition that the products of the two groups are the same; this time, the common product is 36. However, it does not satisfy the condition that the product is equal to the number of rearrangements.